The Combination Contract with the Channel Rebates and Penalty Based on the Return Policy for a Three Level Supply Chain

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Abstract: This paper investigates the coordination of a three-level supply chain with a combination contract which combines the channel rebates and penalty based on the return policy. The supply chain consists of one supplier, one distributor, and one retailer who sells short life cycle products. The demand for the product is stochastic and price-dependent. We show that the three-level supply chain achieves full coordination with the combination contract between each pair of nodes. The conditions to coordinate the supply chain are given in the paper.

Keywords: Combination contract; Return policy; Channel rebate; Supply chain

I. Introduction

A return policy specifies the conditions under which a retailer can return unsold merchandise for a full or partial refund. A channel rebate is a payment from a suppler to a retailer based on sales. Both return policy and channel rebates are widely used in commercial practice. Considering the pricing decision faced by a producer of a commodity with a short shelf or demand life, Pasternack(1985) shows that neither a policy of allowing for unlimited returns at full credit nor one allowing for no returns could be optimal. From then on, many papers about return policy were published. Many of them assume that retail price is determined exogenously. Under the situation that the demand is stochastic and depends on the price of the product, Marvel and Peck(1995), Emmons and Gilbert(1998) respectively demonstrated that a return policy cannot coordinate the supply chain. When demand is influenced by retailer sales effort, Taylor(2001) showed that contracts, such as linear rebate and returns or target rebate alone, cannot achieve coordination in a way that is implementable.

For this reason, some combination contracts are presented. Taylor(2001) properly designed a combination contract of return policy and target rebate, which achieves coordination and a win–win outcome. Considering a one-supplier one-retailer supply chain facing stochastic price-dependent demand, He etal.(2004) developed a model which combined the channel rebate and penalty based on the return policy. The results showed that the combination contract can coordinate the supply chain but the return policy alone cannot. However, for the similar supply chain, Wang etal.(2009) presented a combination contract with the buy-back and the target rebates. They demonstrated that the supply chain can also be coordinated by their combination contract. It is noteworthy that the models mentioned above are all for the two level supply chain.

With respect to a three level supply chain, Ding and Chen(2008) constructed a so-called flexible return policy by setting the rules of pricing while postponing the determination of the final contract parameters. Such policy can be considered as the combination of the return policy and the postponement strategy, with which a three level supply chain can be fully coordinated. In addition, Hou and Qiu(2008) combined the return policy and the profit sharing contract, in which a wholesaler transacts with a retailer by a return policy and a manufacturer transacts with the wholesaler by a profit sharing contract. They showed that a three level supply chain can be coordinated by such a combined contract. Both Ding etal. and Hou etal. considered the demand be stochastic in their model, but not dependent on the price.

As for stochastic price-dependent demand, whether a combined contract with the channel rebates and penalty based on the return policy can coordinate a three-level supply chain or not? In this paper, we present a model to address the problem based on the model of He etal. and that of Wang etal., which are good for two level supply chain as mentioned above.

II. Basic Settings

Consider a three level supply chain consisting of one supplier, one distributor and one retailer. All of them are risk neutral and make decisions according to the expected profit maximization under information sharing. The supplier produces and sells short life cycle products to the distributor. The distributor provides warehousing and logistics services for the product, and then distributes the products to the retailer, who faces uncertain price-dependent demand. The decision procedure of the supply chain is described as follows. At first, the supplier announces the wholesale price w_1 and the return price r_1 of the product to the distributor. Then, the distributor determinates the wholesale price w_2 and the return price r_2 to the retailer. After that, the retailer makes decisions of the order quantity q to the distributor and the retail price p to customers.

The distributor passes the order from the retailer to the supplier. Reversely, the products are passed from the supplier to the retailer via the distributor before the selling season arrives. After the selling season, if there are any products unsold, the retailer returns them to the distributor, then the distributor to the supplier, each at the return price r_2 and r_1 respectively.

The following are parameters and variables used in the model:

c_1: the supplier' cost of unit product
c_2: the distributor's value-added cost of unit product
c_3: the retailer's value-added cost of unit product
c: the total cost of the supply chain and $c = c_1 + c_2 + c_3$
w_1: the supplier's wholesale price to the distributor
w_2: the distributor's wholesale price to the retailer
m_1: the channel rebates per unit over the sales target or the penalty per unit under the sales target set by the supplier to the distributor
m_2: the channel rebates per unit over the sales target or the penalty per unit under the sales target set by the distributor to the retailer
r_1: the return price provided by the supplier to the distributor for unit product unsold
r_2: the return price provided by the distributor to the retailer for unit product unsold
Γ: the sales target set by the supplier for the distributor, also by the distributor for the retailer
p: the retail price of unit product set by the retailer
q: the order quantity from the retailer to the distributor, then to the supplier
x: the stochastic price-dependent demand, which can be decomposed into a non-stochastic, y(p), and a stochastic, δ, in additive, i.e., $x(p,\delta) = y(p) + \delta$, where $y(p)$ is an decreasing function of the retail price p and δ is a random variable over $[A,B]$ with probability density $f()$ and cumulative distribution $F()$
μ: mean of the random variable δ
σ: standard variation of the random variable δ
$S(q,p)$: the expected sales during the selling season
EPS is the expected profit of the supplier. EPD is that of the distributor and EPR is that of the retailer.

In the paper, we assume that

Assumption A1. $w_1 < w_2,\ r_2 \le r_1,\ w_2 + c_3 \le p$,

$$w_1 + c_2 \le w_2,\ v < c_1 \le w_1.$$

Assumption A2. $r_1 + v < w_1 + c_2,\ r_2 + v < w_2 + c_3$.

Assumption A1 reflects that the supplier, the distributor and the retailer are all rational. Assumption A2 avoids that the distributor or the retailer can be involved in arbitrage from the policy provided by the supplier or the distributor.

III. The Basic Model

Given the operating settings described as above, the distributor orders q units of products before the selling season. The retailer's expected sales $S(q,p)$ during the selling season is

$$S(q,p) = \min(x,q) = q - \int_{A+y(p)}^{q} F[x - y(p)]dx \tag{1}$$

The centralized supply chain

When the supply chain is centralized controlled, the supply chain's total expected profit EPT_C is

$$EPT_C = p[q - \int_{A+y(p)}^{q} F(x - y(p))dx] - cq \tag{2}$$

The subscript C stands for the centralized controlled system. Defining (q_C^*, p_C^*) as the optimal order quantity and the optimal retail price, we get that

$$\frac{\partial EPT_C}{\partial q}\Big|_{q=q_C^*} = (p\frac{\partial S(q,p)}{\partial q} - c)\Big|_{q=q_C^*} = 0 \tag{3}$$

and

$$\frac{\partial EPT_C}{\partial p}\Big|_{p=p_C^*} = [S(q,p) + p\frac{\partial S(q,p)}{\partial p}]\Big|_{p=p_C^*} = 0 \tag{4}$$

There is no doubt that the centralized system can get to the maximal expected profit. So it can be regarded as a benchmark for the decentralized supply chain.

The decentralized system with return policy only

For the decentralized system without any contract, in which the demand is stochastic and price-dependent, the retailer, the distributor and the supplier decide independently. Their profit is respectively

$$EPS_{DN} = (w_1 - c_1)q \tag{5}$$

$$EPD_{DN} = (w_2 - w_1 - c_2)q \tag{6}$$

$$EPR_{DN} = pS(q,p) - (w_2 + c_3)q \tag{7}$$

Here, the subscript DN implies the decentralized system without any contract. In the scenario like this, it is well known that the supply chain is inefficient. We directly consider the decentralized system with return policy only. The retailer's expected profit EPR is

$$EPR_{DR} = (p - r_2)S(q,p) - (w_2 + c_3 - r_2)q \tag{8}$$

The subscript DR represents the decentralized system without contract. To maximize his expected profit, the retailer makes up his mind about (q_R^*, p_R^*) according to

$$\frac{\partial EPR_{DR}}{\partial q}\Big|_{q=q_R^*} = [(p - r_2)\frac{\partial S(q,p)}{\partial q} - (w_2 + c_3 - r_2)]\Big|_{q=q_R^*} = 0 \tag{9}$$

and

$$\frac{\partial EPR_{DR}}{\partial p}\Big|_{p=p_R^*} = [S(q,p) + (p - r_2)\frac{\partial S(q,p)}{\partial p}]\Big|_{p=p_R^*} = 0 \tag{10}$$

We suppose that the retailer would maximize the supply chain's total profit. He need to set the order quantity and the retail price to satisfy $q_R^* = q_C^*,\ p_R^* = p_C^*$. From Eqs. (3) and

(9), (4) and (10),we know that $r_2 = 0,\ w_2 = c_1 + c_2$. Taking assumption A1 into consideration, we can deduce that $w_2 = c_2$, $w_1 = c_1$, and $r_1 = 0$. It means that the supplier and the distributor make no profit so as to coordinate the supply chain. As long as the wholesale price of the supplier or the distributor is higher than his cost, the system becomes inefficient. So we might conclude that the supply chain can not get full coordination with the contract of the return policy only.

IV. The Combination Contract

As for the decentralized system facing the stochastic price-dependent demand, the supplier, the distributor and the retailer make decisions independently under the combination contract with the channel rebates and penalty based on the return policy. The profit of the supplier, the distributor, the retailer is respectively

$$EPS_{DC} = (r_1 - m_1)S(q,p) + (w_1 - c_1 - r_1)q + m_1\Gamma \qquad (11)$$

$$\begin{aligned} EPD_{DC} = {} & (r_2 - r_1 + m_1 - m_2)S(q,p) + \\ & (w_2 - w_1 + r_1 - r_2 - c_2)q + (m_2 - m_1)\Gamma \end{aligned} \qquad (12)$$

$$\begin{aligned} EPR_{DC} = {} & pS(q,p) + r_2(q - S(q,p)) + \\ & m_2(S(q,p) - \Gamma) - (w_2 + c_3)q \end{aligned} \qquad (13)$$

The subscript DC implies the decentralized system with the combined contract. The first item of equation (13) is the sales revenue of the retailer. The second is the refund from the distributor for the products unsold and the forth is the cost of the retailer. Please pay attention to the third one. It can be positive or negative. If the expected sales are larger than the target, the third item means the channel rebates. However it implies the penalty if the expected sales is less than the target. This is the reason that we call our contract the combined contract with the channel rebate and penalty based on the return policy. The retailer determines the optimal order quantity and the retail price (q_R^*, p_R^*) to maximize his profit. Hence, (q_R^*, p_R^*) should satisfy

$$\frac{\partial EPR_{DC}}{\partial q}\bigg|_{q=q_R^*} = [(p - r_2 + m_2)\frac{\partial S(q,p)}{\partial q} + r_2 - c_3 - w_2]\bigg|_{q=q_R^*} = 0 \qquad (14)$$

and

$$\frac{\partial EPR_{DC}}{\partial p}\bigg|_{p=p_R^*} = [(p - r_2 + m_2)\frac{\partial S(q,p)}{\partial p} + S(q,p)]\bigg|_{p=p_R^*} = 0 \qquad (15)$$

In order to achieve the maximal profit of the whole supply chain, (q_R^*, p_R^*) should be equal to (q_C^*, p_C^*). Combining equation (3) with (14) and equation (4) with (15), we have

$$\begin{cases} r_2 = m_2 \\ m_2 = w_2 - c_1 - c_2 \end{cases} \qquad (16)$$

Similarly, to coordinate the supplier and the distributor, we require

$$\begin{cases} r_1 = m_1 \\ m_1 = w_1 - c_1 \end{cases} \qquad (17)$$

Then, based on Eqs.(11),(12),(13),(16) and (17), the expected profit of the supplier, the distributor, the retailer is respectively

$$EPS_{DC} = m_1\Gamma \qquad (18)$$

$$EPD_{DC} = (m_2 - m_1)\Gamma \qquad (19)$$

$$EPR_{DC} = EPT_C - m_2\Gamma \qquad (20)$$

Obviously, the sum of the expected profit of the supplier, the distributor and the retailer is equal to that of the centralized controlled supply chain *EPT*. So we conclude that the combination contract makes the supply chain get to the maximal profit of the system. Furthermore, it should be guaranteed that the profit of the supplier, the distributor, the retailer is respectively larger than that in the decentralized system without any contract. Therefore, the value of $m_1(r_1)$, $m_2(r_2)$ and Γ should be carefully selected. From Eqs.(5)-(7) and (18)-(20), we deduced that

$$(w_1 - c_1)q_{DN}^* \le m_1\Gamma \le EPT_C - EPR_{DN} - (w_2 - c_1 - c_2)q_{DN}^* \qquad (21)$$

and

$$(w_2 - c_1 - c_2)q_{DN}^* \le m_2\Gamma \le EPT_C - EPR_{DN} \qquad (22)$$

Thus, to sum up the above arguments, conditions to coordinate the three-level supply chain fully are

$$\begin{cases} r_1 = m_1 \\ m_1 = w_1 - c \\ r_2 = m_2 \\ m_2 = w_2 - c_1 - c_2 \\ (w_1 - c_1)q_{DN}^* \le m_1\Gamma \le EPT_C - EPR_{DN} - (w_2 - c_1 - c_2)q_{DN}^* \\ (w_2 - c_1 - c_2)q_{DN}^* \le m_2\Gamma \le EPT_C - EPR_{DN} \end{cases} \qquad (23)$$

V. Conclusion

In this paper, we study combination contract with the channel rebates and penalty based on the return policy for a three level supply chain, facing stochastic price-dependent demand. The combination contract presented in this paper can coordinate the three level supply chain fully. However, both the channel rebate and the sales target should be carefully selected. The conditions, which can achieve full coordination, are deduced in the paper.

The analysis in this paper can be extended to the scenario in which multi-products are sold. More research can also be carried on to the three level supply chain, consisting of one supplier, one distributor and multi-retailers.

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